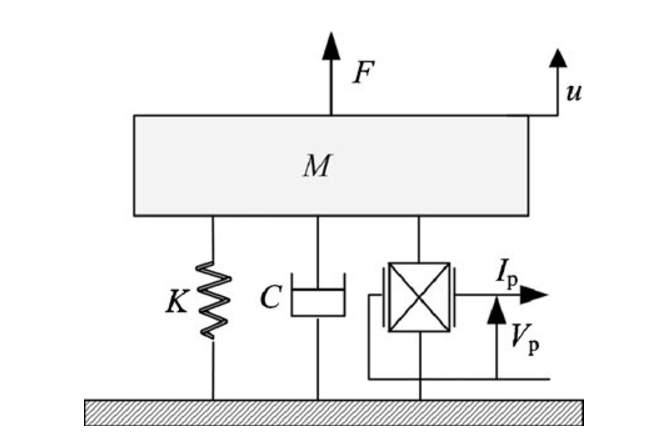
Analytical Derivation of Lumped model of Piezo energy harvester

The proposed system of meandered piezo energy harvester is a combination of the single and coupled cantilever elements. The individual cantilever elements with proof mass can be represented by the equivalent model shown in Fig. 1. The coupled cantilever element with proof mass can be represented by the equivalent model as shown in Fig. 2.

  
Fig. 1. Equivalent model of the energy harvesting device. This figure was uploaded by Liya Zhu. Ref - **A new synchronized switching harvesting scheme employing current doubler rectifier. (Research Gate)**

The complete analytical derivation of 1 DOF lumped model.

Considering the element parameters,   
m1 = Proof-Mass  
u0 = Base Excitation displacement  
u1 = Relative Displacement of Proof Mass  
k1 = Stiffness Parameter  
c1 = Damping Parameter  
α1 = Force Factor of PZT  
Cp1 = Clamped capacitance of PZT   
R = Load Resistance  
V1 = Generated potential across R

The Electro-mechanical system in governed by the modified Duffing Equations for a single piezo-electric oscillator as shown:

eq. (1) eq. (2)

Where, , and represent the 1st , 2nd derivative of relative displacement and 1st derivative of Voltage generated respectively.

Taking Laplace transform of both eq. (1) and eq. (2) we have;

m1.u1.s2 = -k1 (u1 – u0) – c1(u1 – u0)s – α1.V1 and,

V1/R = α1(u1 – u0)s – Cp1.V1.s

Further simplifying the equations we have,

(m1.s2 + k1 + c3.s). u1 + α1.V1 = (k1 + c1.s).u0  eq. (3)

α1.u1.s – (Cp1.s + 1/R) V1 = α1.u0.s eq. (4)

Therefore, we have a system of equations in 2 unknowns u1 and V1. This system of equation cane be transformed into the matrix form as follows:

= u0

Substituting s = j.ω, where j = and ω = 2*f* is the angular frequency.

= u0 eq. (6)

Further upon simplifying the above system of equations we get;

Let’s say, [A].[X] = [B], where

=

= u0

=

The above system of equations has ben solved by using matrix operations in the following manner.  
We know, if [A][X] = [B],   
then, [X] = [A]-1.[B]  
and [A]-1 = Adj [A]/|A|

Therefore, we have,

eq. (7)

We know, Adj[A] = Transpose of co-factor matrix of A.

Cofactor of Minor [A] =  ;

Adj [A] =

Hence we have,

[A]-1 =

Upon Solving [X] = [A]-1[B]. we have

= u0. eq. (8)

Hence,

= ;

=

The above system of equation is then fed into MATLAB script and simulated across a wide range of frequency for performance analysis.

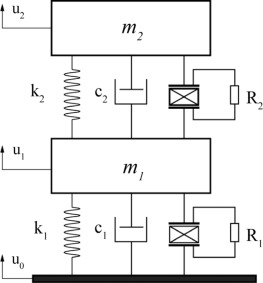


Fig. 2. This figure was uploaded by Han Xiao. Ref - **A multi-degree of freedom piezoelectric vibration energy harvester with piezoelectric elements inserted between two nearby oscillators. (Elsevier)**

The complete analytical derivation of 2 DOF lumped model.

Considering the element parameters,   
m1 = Proof-Mass 1  
m2 = Proof-Mass 2  
u0 = Base Excitation displacement   
u1 = Relative Displacement of Proof Mass 1  
u2 = Relative Displacement of Proof Mass 2  
k1 = Stiffness Parameter 1  
k2 = Stiffness Parameter 2  
c1 = Damping Parameter 1  
c2 = Damping Parameter 2  
α1 = Force Factor of PZT 1  
α2 = Force Factor of PZT 2  
Cp1 = Clamped capacitance of PZT 1  
Cp2 = Clamped capacitance of PZT 2  
R = Load Resistance  
V1 = Generated potential across R  
V2 = Generated potential across R

The Electro-mechanical system in governed by the modified Duffing Equations for a 2-DOF piezo-electric oscillator as shown:

eq. (1)

eq. (2)

eq. (3)

eq. (4)

Now, applying Laplace equation to all the equations, we have;

Further simplifying the equations, we get;

).

).

We have a system of equations in 4 unknown variables, namely u1, u2, V1, V2 ;

Converting the system of equation into matrix form [A].[X] = [B]

. =

Substituting s = j.ω, where j = √(-1) and ω = 2πf is the angular frequency.

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